STAT20 Homework #8

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Table of Contents

### Introduction

This is Homework #8, which contains questions from Chapters 26 & 27. *Due 9 April 2021.*

# Chapter 26

### 1. Loosely based on Ch 26 F4:

A scale is calibrated using a weight that is known to be exactly 1 kg. Each data set below represents repeated measurements of the weight. Assume the Gauss model (measurement = true weight of 1 kg + measurement error + bias), with measurement errors following the normal curve. Since the weight actually weighs 1 kg, the null hypothesis says that the expected value should be 1 kg (no bias). For each dataset below, make a t-test to see whether the scale is properly calibrated or not. In one case, this is impossible. Which one, and why? The data is listed as the amount above or below 1kg, and the units are micrograms.

##### a) 1, -2, 9

aWeight = c(1, -2, 9)  
t.test(aWeight, mu = 1)

##   
## One Sample t-test  
##   
## data: aWeight  
## t = 0.50767, df = 2, p-value = 0.6621  
## alternative hypothesis: true mean is not equal to 1  
## 95 percent confidence interval:  
## -11.45874 16.79207  
## sample estimates:  
## mean of x   
## 2.666667

In this scenario, it is still possible for the scale to be properly calibrated. 95% confidence interval includes the mean value that it should have.

##### b) 1, -2, 9, 14, 8, 15, -1

bWeight = c(1, -2, 9, 14, 8, 15, -1)  
t.test(bWeight, mu = 1)

##   
## One Sample t-test  
##   
## data: bWeight  
## t = 1.993, df = 6, p-value = 0.09333  
## alternative hypothesis: true mean is not equal to 1  
## 95 percent confidence interval:  
## -0.2039156 12.7753441  
## sample estimates:  
## mean of x   
## 6.285714

In this scenario, it is still possible for the scale to be properly calibrated, however it is much less likely. 95% confidence interval includes the mean value that it should have.

##### c) 1

It is impossible to commit a t-test with this data as there is no way to construct a 95% interval with only 1 data value.

##### d) 1, 14

dWeight = c(1, 14 )  
t.test(dWeight, mu = 1)

##   
## One Sample t-test  
##   
## data: dWeight  
## t = 1, df = 1, p-value = 0.5  
## alternative hypothesis: true mean is not equal to 1  
## 95 percent confidence interval:  
## -75.09033 90.09033  
## sample estimates:  
## mean of x   
## 7.5

In this scenario, it is still possible for the scale to be properly calibrated, but there should be more samples taken. 95% confidence interval includes the mean value that it should have.

### 2. Ch 26 Rev 8:

Bookstores like education, one reason being that educated people are more likely to spend money on books. National data show the nationwide average educational level to be 13 years of schooling completed, with an SD of about 3 years, for persons age 18 and over. A bookstore is doing a market survey in a certain county, and takes a simple random sample of 1,000 people age 18 and over. They find the average educational level to be 14 years, and the SD is 5 years. Can the difference in average educational level between the sample and the nation be explained by chance variation? If not, what other explanation can you give?

##### Explanation

educationSample = 14  
nationSD = 3  
educationDraws = 1000   
educationSE\_sum = sqrt(educationDraws) \* nationSD # = 94.86833  
educationSE\_avg = educationSE\_sum/educationDraws # = 0.09486833  
  
nationMean = 13  
educationZ = (educationSample - nationMean)/educationSE\_avg # 10.54093  
  
(1 - pnorm(10.54093)) \* 100

## [1] 0

In conclusion, this result is so close to 0 that it can not be due to chance and in this scenario the null hypothesis is rejected. However an alternate explanation would be that the area that is chosen has a higher average education than the rest of the country and there are other areas with lower average education to even it out.

### 3. Ch 26 Rev 11:

According to the census, the median household income in Atlanta (1.5 million households) was $52,000 in 1999. In June 2003, a market research organization takes a simple random sample of 750 households in Atlanta; 56% of the sample households had incomes over $52,000. Did median household income in Atlanta increase over the period 1999 to 2003?

##### a) Formulate null and alternative hypotheses in terms of a box model. (It’s a little tricky, think about what percent you should expect the 56% to be if the median household income didn’t change.)

null = The percentage drawn is due to chance and median household income has not changed since 1999.

alt = Median household income has increased to over $52,000 since 1999.

##### b) Calculate the appropriate test statistic and P.

atlantaObserved = 750\*.56 # = 420  
atlantaDraws = 750  
  
atlantaEV\_sum = 750\*.50 # = 375  
atlantaSD = (1-0) \* sqrt(0.5 \* 0.5) # = 0.5  
atlantaSE\_sum = sqrt(atlantaDraws) \* atlantaSD # = 13.69306  
  
atlantaZ = (atlantaObserved - atlantaEV\_sum)/atlantaSE\_sum # = 3.286335  
  
(1 - pnorm(atlantaZ)) \* 100

## [1] 0.05075005

##### c) Did median family income go up?

Since the p-value is quite small this number is highly significant, meaning that we reject the null hypothesis and it is most likely not due to chance. Meaning that the average value income in a home has likely gone up from $52,000 since 1999.

# Chapter 27

### 4. Ch 27 A6:

One hundred draws are made at random with replacement from box F: the average of these draws is 51 and their SD is 3. Independently, 400 draws are made at ran-dom with replacement from box G: the average of these draws is 48 and their SD is 8. Someone claims that both boxes have the same average. What do you think? You don’t need to do a formal hypothesis test, but think about it in that way.

##### a) What do you think?

Given the number of draws and the average and SD gotten from these draws I find it unlikely that the means would be the same. Without doing a formal hypothesis test we can find that just by looking at the SEs between the two

boxfSE\_sum = sqrt(100) \* 3 # = 30  
boxfSE\_avg = boxfSE\_sum/100 # = 0.3  
  
boxgSE\_sum = sqrt(400) \* 8 # = 160  
boxgSE\_avg = boxgSE\_sum/400 # = 0.4

That it is quite unlikely to have a difference in the average draw sum of 3 between the two boxes meaning that it is quite unlikely that they have the same mean.

### 5. Ch 27 B3:

In 1970, 59% of college freshmen thought that capital punishment should be abolished; by 2005, the percentage had dropped to 35%. Is the difference real, or can it be explained by chance? You may assume that the percentages are based on two independent simple random samples, each of size 1,000. For this one, do a formal hypothesis test.

##### Hypothesis test

percent1970 = 0.59  
percent2005 = 0.35  
collegeSample = 1000  
  
SD\_1970 = (1 - 0) \* sqrt(0.59 \* 0.41) # = 0.4918333  
SE\_1970\_sum = sqrt(collegeSample) \* SD\_1970 # = 15.55313  
SE\_1970\_avg = (SE\_1970\_sum/collegeSample) \* 100 # = 1.555313%  
  
SD\_2005 = (1 - 0) \* sqrt(0.35 \* 0.65) # = 0.4769696  
SE\_2005\_sum = sqrt(collegeSample) \* SD\_2005 # = 15.0831  
SE\_2005\_avg = (SE\_2005\_sum/collegeSample) \* 100 # = 1.50831%  
  
SE\_1970\_2005\_diff = sqrt(SE\_1970\_avg^2 \* SE\_2005\_avg^2) # = 2.345895  
  
collegeMean = 0  
collegeDiff = (percent1970 - percent2005) \* 100 # = 24%  
collegeZ = (collegeDiff - collegeMean)/SE\_1970\_2005\_diff # = 10.23064  
  
(1 - pnorm(10.23064)) \* 100

## [1] 0

Since the p-value is about 0 it is enough evidence to reject the null hypothesis. This difference in results is not due to chance. The number of college students that want to abolish capital punishment has decreased since 1970.

### 6. Ch 27 C2:

(Hypothetical.) Is Wheaties a power breakfast? A study is done in an elementary statistics class; 499 students agree to participate. After the midterm, 250 are randomized to the treatment group, and 249 to the control group. The treatment group is fed Wheaties for breakfast 7 days a week. The control group gets Sugar Pops.

##### a) Final scores averaged 66 for the treatment group; the SD was 21. For the control group, the figures were 59 and 20. What do you conclude?

wheaties\_avg = 66  
wheaties\_SD = 21  
wheaties\_size = 250  
wheaties\_SE\_sum = sqrt(wheaties\_size) \* wheaties\_SD # = 332.0392  
wheaties\_SE\_avg = wheaties\_SE\_sum/wheaties\_size # = 1.328157  
  
sugarpops\_avg = 59  
sugarpops\_SD = 20  
sugarpops\_size = 249  
sugarpops\_SE\_sum = sqrt(sugarpops\_size) \* sugarpops\_SD # = 315.5947  
sugarpops\_SE\_avg = sugarpops\_SE\_sum/sugarpops\_size # = 1.267449  
   
SE\_wheaties\_sugarpops\_diff = sqrt(wheaties\_SE\_avg^2 + sugarpops\_SE\_avg^2) # = 1.835872  
elementaryDiff = (wheaties\_avg - sugarpops\_avg) # 7  
elementaryZ = (elementaryDiff - 0)/SE\_wheaties\_sugarpops\_diff # = 3.812902  
  
(1 - pnorm(elementaryZ)) \* 100

## [1] 0.006867225

##### b) What aspects of the study could have been done “blind?”

The blindness could have been done by not letting the patients know which group they were in (treatment/control). But, this would be extremely difficult as you can not control which of the students recognized the specific cereal. However, the person that was in charge of receiving this data could have been blind to affect it even less.

### 7. Ch 27 D4:

Many observational studies conclude that low-fat diets protect against cancer and cardiovascular “events” (heart attacks, stroke, and so forth). Experimental results, however, are generally negative. In 2006, the Women’s Health Initiative (WHI) published its results. This was a large-scale randomized trial on women who had reached menopause. As one part of the study, 48,835 women were randomized: 19,541 were assigned to the treatment group and put on a low-fat diet. The other 29,294 women were assigned to the control group and ate as they normally would. Subjects were followed for 8 years. Among other things, the investigators found that 1,357 women on the low-fat diet experienced at least one cardiovascular event, compared to 2,088 in the control group. Can the difference between the two groups be explained by chance? What do you conclude about the effect of the low-fat diet?

##### Conclusion

menopauseT\_sum = 1357  
menopauseT\_size = 19541  
menopauseT\_percent = menopauseT\_sum/menopauseT\_size # 0.06944373  
menopauseT\_SD = (1 - 0) \* sqrt(menopauseT\_percent \* (1 - menopauseT\_percent)) # 0.2542072  
menopauseT\_SE\_sum = sqrt(menopauseT\_size) \* menopauseT\_SD # = 35.5354  
menopauseT\_SE\_avg = menopauseT\_SE\_sum/menopauseT\_size # = 0.001818505  
  
menopauseC\_sum = 2088  
menopauseC\_size = 29294  
menopauseC\_percent = menopauseC\_sum/menopauseC\_size # 0.07127739  
menopauseC\_SD = (1 - 0) \* sqrt(menopauseC\_percent \* (1 - menopauseC\_percent)) # 0.2572876  
menopauseC\_SE\_sum = sqrt(menopauseC\_size) \* menopauseC\_SD # = 44.03604  
menopauseC\_SE\_avg = menopauseC\_SE\_sum/menopauseC\_size # = 0.001503244  
   
SE\_menopauseT\_menopauseC\_diff = sqrt(menopauseT\_SE\_avg^2 + menopauseC\_SE\_avg^2) \* 100 # = 0.2359386  
womenDiff = (menopauseT\_percent - menopauseC\_percent) \* 100 # -0.1833661  
womenZ = (womenDiff - 0)/SE\_menopauseT\_menopauseC\_diff # = -0.007771771  
  
pnorm(womenZ) \* 100

## [1] 21.85271

Yes, it is most likely to be due to just chance. In this scenario you do not reject the null hypothesis and having a low-fat diet does not have a significant effect on cardiovascular diseases.

### 8. Ch 27 Rev 3:

The Gallup poll asks respondents how they would rate the honesty and ethical standards of people in different fields-very high, high, average, low, or very low. The percentage who rated clergy “very high or high” dropped from 60% in 2000 to 54% in 2005. This may have been due to scandals involving sex abuse; or it may have been chance variation. (You may assume that in each year, the results are based on independent simple random samples of 1,000 persons in each year.)

##### a) Should you make a one-sample z-test or a two-sample z-test? Why?

Two-sample z test to compare the different percentage rates depending on the years especially because it is given that they are independant.

##### b) Formulate the null and alternative hypotheses in terms of a box model. Do you need one box or two? Why? How many tickets go into each box? How many draws? What do the tickets show? What do the null and alternative hypotheses say about the box(es)?

null = The number of people who rate clergy as very high or high has not changed from 2000 to 2005.

alt = The number of people who rate clergy as very high or high has dropped from 2000 to 2005.

size = The total number of respondents, unknown.

draws = 1000 for each year

##### c) Can the difference between 60% and 54% be explained as a chance variation? Or was it the scandals? Or something else?

clergy2000\_avg = 0.60  
clergy2000\_size = 1000  
clergy2000\_SD = (1 - 0) \* sqrt(clergy2000\_avg \* (1 - clergy2000\_avg)) # 0.4898979  
clergy2000\_SE\_sum = sqrt(clergy2000\_size) \* clergy2000\_SD # = 15.49193  
clergy2000\_SE\_avg = clergy2000\_SE\_sum/clergy2000\_size # = 0.01549193  
  
clergy2005\_avg = 0.54  
clergy2005\_size = 1000  
clergy2005\_SD = (1 - 0) \* sqrt(clergy2005\_avg \* (1 - clergy2005\_avg)) # 0.4983974  
clergy2005\_SE\_sum = sqrt(clergy2005\_size) \* clergy2005\_SD # = 15.76071  
clergy2005\_SE\_avg = clergy2005\_SE\_sum/clergy2005\_size # = 0.01576071  
   
SE\_clergy2000\_clergy2005\_diff = sqrt(clergy2000\_SE\_avg^2 + clergy2005\_SE\_avg^2) \* 100 # = 2.209977  
clergyDiff = (clergy2000\_avg - clergy2005\_avg) \* 100 # 6  
clergyZ = (clergyDiff - 0)/SE\_clergy2000\_clergy2005\_diff # = 2.71496  
  
(1 - pnorm(clergyZ)) \* 100

## [1] 0.331419

It is unlikely to be due to chance variation. The number of people that think of clergys as very high or high has dropped since the year 2000. In this scenario we should reject the null hypothesis.

### 9. Ch 27 Rev 4:

This continues the previous exercise. In 2005, 65% of the respondents gave medical doctors a rating of “very high or high,” compared to a 67% rating for druggists. Is the difference real, or chance variation? Or do you need more information to decide? If the difference is real, how would you explain it? Discuss briefly. You may assume that the results are based on a simple random sample of 1,000 persons taken in 2005; each respondent rated clergy, medical doctors, druggists, and many other professions.

##### Discuss Briefly

doctor2005\_avg = 0.65  
doctor2005\_size = 1000  
doctor2005\_SD = (1 - 0) \* sqrt(doctor2005\_avg \* (1 - doctor2005\_avg)) # 0.4769696  
doctor2005\_SE\_sum = sqrt(doctor2005\_size) \* doctor2005\_SD # = 15.0831  
doctor2005\_SE\_avg = doctor2005\_SE\_sum/doctor2005\_size # = 0.0150831  
  
druggist2005\_avg = 0.67  
druggist2005\_size = 1000  
druggist2005\_SD = (1 - 0) \* sqrt(druggist2005\_avg \* (1 - druggist2005\_avg)) # 0.4702127  
druggist2005\_SE\_sum = sqrt(druggist2005\_size) \* druggist2005\_SD # = 14.86943  
druggist2005\_SE\_avg = druggist2005\_SE\_sum/druggist2005\_size # = 0.01486943  
   
SE\_doctor2005\_druggist2005\_diff = sqrt(doctor2005\_SE\_avg^2 + druggist2005\_SE\_avg^2) \* 100 # = 0.02242772  
dDiff = (doctor2005\_avg - druggist2005\_avg) \* 100 # -2  
dZ = (dDiff - 0)/SE\_doctor2005\_druggist2005\_diff # = -0.8917537  
  
pnorm(dZ) \* 100

## [1] 17.25135

This is most certainly due to chance variation. There is no reason to reject the null hypothesis in this scenario as it is quite a high probability to occur. The rates between the two are most likely the same to one another but it was just a bad sample.

### 10. Ch 27 Rev 7:

When convicts are released from prison, they often return to crime and are arrested again (recidivism). The Department of Labor ran a randomized controlled experiment to find out if providing income support to ex-convicts during the first months after their release reduces recidivism. The experiment was done on a group of convicts being released from prisons in Georgia. Income support was provided for the treatment group, like unemployment insurance, and the control group received no payment.

##### a) 592 prisoners were assigned to the treatment group, and of them 48.3%were rearrested within a year of release. 154 were assigned to the control group, and of them 49.4% were rearrested within a year of release. Did income support reduce recidivism? Answer yes or no, and explain briefly.

prisonerT\_avg = 0.483  
prisonerT\_size = 592  
prisonerT\_SD = (1 - 0) \* sqrt(prisonerT\_avg \* (1 - prisonerT\_avg)) # 0.4997109  
prisonerT\_SE\_sum = sqrt(prisonerT\_size) \* prisonerT\_SD # = 12.15849  
prisonerT\_SE\_avg = (prisonerT\_SE\_sum/prisonerT\_size) \* 100 # = 2.053799  
  
prisonerC\_avg = 0.494  
prisonerC\_size = 154  
prisonerC\_SD = (1 - 0) \* sqrt(prisonerC\_avg \* (1 - prisonerC\_avg)) # 0.499964  
prisonerC\_SE\_sum = sqrt(prisonerC\_size) \* prisonerC\_SD # = 6.20439  
prisonerC\_SE\_avg = (prisonerC\_SE\_sum/prisonerC\_size) \* 100 # = 4.028825  
  
SE\_prisonerT\_prisonerC\_diff = sqrt((2.053799 \* 2.053799) + (4.028825 \* 4.028825)) # = 4.522115  
prisonerDiff = (prisonerT\_avg - prisonerC\_avg) \* 100 # -1.1  
prisonerZ = (prisonerDiff - 0)/SE\_prisonerT\_prisonerC\_diff # = -0.243249  
  
(1 - pnorm(dZ)) \* 100

## [1] 82.74865

In this scenario the p-value is not small and allows for the assumption for it to be due to chance error. There is no reason to reject the null hypothesis. It also implies that income support did not reduce recidivism.

##### b) In the first year after their release from prison, those assigned to the treatment group averaged 16.8 weeks of paid work; the SD was 15.9 weeks. For those assigned to the control group, the average was 24.3 weeks; the SD was 17.3 weeks. Did income support reduce the amount that the ex-convicts worked? Answer yes or no, and explain briefly.

workprisonerT\_avg = 16.8  
workprisonerT\_size = 592  
workprisonerT\_SD = 15.9  
workprisonerT\_SE\_sum = sqrt(workprisonerT\_size) \* workprisonerT\_SD # = 386.8637  
workprisonerT\_SE\_avg = (workprisonerT\_SE\_sum/workprisonerT\_size) # = 0.653486%  
  
workprisonerC\_avg = 24.3  
workprisonerC\_size = 154  
workprisonerC\_SD = 17.3  
workprisonerC\_SE\_sum = sqrt(workprisonerC\_size) \* workprisonerC\_SD # = 214.6874  
workprisonerC\_SE\_avg = (workprisonerC\_SE\_sum/workprisonerC\_size) # = 0.653486  
  
SE\_workprisonerT\_workprisonerC\_diff = sqrt((workprisonerT\_SE\_avg^2) + (workprisonerC\_SE\_avg^2)) # = 1.539638  
workprisonerDiff = (workprisonerT\_avg - workprisonerC\_avg) # -7.5  
workprisonerZ = (workprisonerDiff - 0)/SE\_workprisonerT\_workprisonerC\_diff # = -4.871275  
  
pnorm(workprisonerZ) \* 100

## [1] 5.54403e-05

In this scenario the p-value is very small (about 0). This implies that we reject the null hypothesis and that it is not due to chance erroing meaning that this supports that income support reduced the amount that the ex-convicts worked.